MATH 410 Homework #1
(Due Wednesday, 18 February 2015)

Read: Dobrow Chapter 1, §§1–3; Appendix B, §§B.1–B.4

Problems: page 36, # 1.3, 1.5, 1.8, 1.16, 1.17, 1.26.

1. Recall from class that in a sequence of coin tosses $P(\text{HTH before HHH}) = 3/5$. If you lost money to me on this game, you might want to take HTH next time. I will happily let you have this, but I will then bet that HHT will appear first. Show that this also gives me a favorable bet.

Amazingly, no matter which triplet you pick, I can find one that has a better than even chance of appearing first: If you pick $xx_-$ (i.e., first two the same), then I will pick $yxx$; If you pick $xy_-$ (first two the different), then I will pick $xxy$. Show that this always gives me a favorable bet. By symmetry, it suffices to consider $x = H$ and $y = T$, so there are only four cases. As in the class examples, you need to be alert for points at which either the process restarts, or at which one triplet is certain to appear first in what follows.

2. Let $X$ be your net from a $\$1$ bet on red at roulette. For $i = 1, 2, 3$, let $C_i$ be the event that a column $i$ number is spun, and $G$ be the event that a green number is spun.

(a) Find $E[X \mid C_i]$ for each $i$, and $E[X \mid G]$.

(b) Use the Law of Total Expectation to compute $E[X]$. (Of course, this is the long way to do a problem for which you already know the answer is $-1/19$.)

3. The R command `rbinom(10000, 20, 0.3)` will simulate observations of a binomial random variable $X$ with $n = 20$ and $p = 0.3$. Assign these to a vector called ‘binsim’. Use the simulated values to estimate $P(X \leq 5)$, $P(X = 5)$ and $E[X]$. (Hint: use `mean()`). How do the values compare with theory?