Math 410 Homework #2
(Due Wednesday, 25 February 2015)

Read: Dobrow Chapter 1, §§4-5 (up to conditional variance)
        Chapter 2, §§1-4.

Problems: p. 36, #1.13, 1.24; p. 71, #2.1, 2.4, 2.6, 2.10, 2.20.

• In problem 2.1, parts (a), (b) do not depend on $\alpha$. (Why?)

• For 2.6, I think it is more natural to consider the chain starting at $X_1 \sim \text{Unif}\{1, 2, 3, 4\}$. You can answer 2.6b without matrix machinery.

• If you think like Fermat, you can actually answer 2.10b without matrix machinery, though you may want to check by calculating.

1. Refer to your handout from class showing the joint distribution of $(X, Y)$, where $X$ is the roll of a fair die and $Y$ is the number of heads in $X$ tosses of a fair coin. We saw that $E[Y] = 7/4$.
   (a) Find $\text{Var}(Y)$ directly from the marginal distribution on the handout.
   (b) Find $\text{Var}(Y)$ using the random sum formula by expressing $Y$ as a sum of indicator random variables.

2. Inventory chain. An electronics store sells a video game system. If, at the end of the day, the number of units they have on hand is 1 or 0, they order enough units to bring their total on hand up to 4. In the language of operations research, this is an example of an $(s, S)$ inventory control policy, with $s = 1$ and $S = 4$.
   Suppose that orders are filled by overnight delivery, so that new merchandise arrives before the story opens on the next day. Let $X_n$ be the number of units on hand at the end of the $n$th day. Assume that the number of customers who want to buy a game system each day is 0, 1, or 2 with probabilities 0.5, 0.3, and 0.2, respectively.
   Specify the transition matrix for a Markov chain model of $\{X_n\}$. 